

How High Can You Jump On Mars? (A Lesson In High School Algebra)

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October 11, 2016

Phun with Physics

This article uses nothing more than high school algebra to show some of the astounding bits of knowledge that can be had just by moving some simple equations around. Oh, you're not a math freak? That's OK. Skip over the math and just read the words. You'll be able to understand what is happening. My goal here is not to force you to read through a lot of mathematics, but rather to show you how one can start with a couple of discoveries about natural law and apply a little math logic to reach some interesting conclusions. Read on and be amazed!

Newton II

If you could only ever know one single law of physics, it shouldn't be Einstein's famous $E = mc^2$. It should be Newton's second law of motion, which actually affects just about everything that goes on in your life. Newton's second law says that if you apply a force to a massive object (in the absence of friction, etc.), it will accelerate in the direction of the applied force, and the rate of acceleration will be the amount of force applied divided by the mass of the object:

$$\textit{Acceleration of the object} = \textit{Force applied to the object} / \textit{Mass of the object}$$

In mathematical shorthand, this is written

$$a = F/m$$

(If you multiply both sides of this equation by the mass m , the two sides remain equal, and you get

$$F = ma$$

This is the way most physicists remember Newton's second law.)



Figure 1: The planet Mars

Gravity

As if this weren't enough, Newton also discovered the law of gravitation, which describes the force of gravitational attraction between two masses. What he discovered was that if you were to place two masses, say M and m a distance d apart (measured from their centers), each mass will be attracted toward the other with a gravitational force given by

$$F = \frac{GMm}{d^2}$$

where G is a universal constant. So, if M is the mass of the Earth and m is your mass, we can use these two laws to determine what your acceleration would be if you were falling near the surface of the Earth. Just substitute the expression for the gravitational force into Newton's second law:

$$F = \frac{GMm}{d^2} = ma$$

Well, would you look at that! Your mass is on both sides of the equation. We can divide both sides of the equation by your mass and what remains will still be an equality (Remember your high school algebra?) So your acceleration due to the force of Earth's gravity doesn't depend on your mass at all:

$$a = \frac{GM}{d^2}$$

It only depends on the mass of the Earth and how far you are from the center of the Earth. Well, if you are falling near the surface of the Earth, then the value of the distance d between you and the center of the Earth is essentially the radius of the Earth, which is about 4,000 miles. If you look up the mass of Earth and the value of the universal constant G , you can work this out, and you would find that the acceleration you compute is about 32 feet per second per second. Physicists usually use the symbol g to represent this value of acceleration. So, when you hear that some jet fighter pilot experienced an acceleration of 4 g 's during a turn, you now know that she was accelerating at a rate of 4 x 32 feet per second per second.

Calculating the Mass of the Earth

But how do we know the mass of the Earth? Take a good look at that equation for gravitational acceleration. Your mass cancelled out. Suppose we ask about the acceleration of the moon toward the center of the Earth under the influence of Earth's gravity. Suppose we let m be the mass of the moon and d be the distance from the center of the Earth to the center of the moon. Just as we saw for your mass, the acceleration of the moon depends on the mass of the Earth, not the mass of the moon. Can we measure the acceleration experienced by the moon as it moves in its circular orbit around the Earth? You betcha! Then we can use our high school algebra to solve our acceleration equation for the mass of the Earth:

$$M = \frac{ad^2}{G}$$

But it's even easier than this. We don't have to measure the acceleration of the moon revolving around the Earth. I won't show it here, but it is easy to show that the acceleration of any object moving uniformly around a circular path is given by

$$a = \frac{v^2}{R}$$

where v is the linear speed of the object along the circular path and R is the radius of the circle. Remembering Newton's second law, we can multiply both sides of this equation by the mass of the object to find the force being applied to the object:

$$F = ma = \frac{mv^2}{R}$$

This is the expression for centrifugal force, that strong pull you feel when you are on a fast-moving merry-go-round.

But it's even easier than this. The velocity of an object moving in a circle is just the circumference of the circle (distance traveled) divided by the time needed to make a full circle. So the velocity of the moon along its orbit is

$$v = \frac{2\pi R}{T}$$

where T is the period of the orbit. For the moon, the period is about 28 days. Substituting this into the equation for acceleration gives

$$a = \frac{4\pi^2 R^2}{T^2 R} = \frac{4\pi^2 R}{T^2}$$

where I used the last step to cancel out the extra R that appeared in both the numerator and the denominator. Now we can substitute this to get the mass of the Earth in terms of quantities that are easy to measure:

$$M = \frac{ad^2}{G} = \frac{aR^2}{G} = \frac{4\pi^2 R^3}{T^2 G}$$

(Here we used the fact that $d = R$, the distance between the center of the moon and the center of the Earth is the same thing as the radius of the moon's orbit.) Amazing! All we need to know to measure the mass M of the Earth is the radius R of the moon's orbit, the period T of the moon's orbit, and the universal constant G . Let's hear a big round of applause for Isaac Newton!

Mars has a couple of small moons, Deimos and Phobos. We can measure their orbits. Therefore, we can calculate the mass of the planet Mars in just the same way we calculated the mass of Earth. Just in case this may come in handy.

Conservation of Energy

When you are relaxing in your easy chair, you are conserving your energy. But in physics the principle of conservation of energy means that the total amount of energy in a closed system is always the same. When it comes to jumping on Earth or on Mars, there are two types of energy that need to be considered. The first is *kinetic energy*, the energy of motion. A mass m moving with a velocity v has a kinetic energy

$$\text{Kinetic Energy} = \frac{mv^2}{2}$$

The other type of energy we need to know about is *potential energy*. The force of gravity acting on your body endows you with potential energy. Your gravitational potential energy is given by

$$\text{Potential Energy} = mgh$$

where m is your mass, g is the acceleration of gravity, and h is your height above the ground. If you are lying on the ground, then $h = 0$ and you have zero potential energy. You may feel like that in bed sometimes, but remember, you could still roll out of the bed and fall to the floor. Lying there in bed, you may not have much kinetic energy, but you still have some potential energy.

But as far as gravity is concerned your total energy is conserved. It remains constant. If you jump, you will start off with zero potential energy (relative to the ground) and as great an initial velocity as the power of your legs can provide. As you move upward through the air, gravity will provide negative acceleration to slow you down and pull you back to earth. At the highest point of your jump, your body will be at its maximum height (h) and will momentarily have zero velocity. The conservation of energy just says the your initial kinetic energy is equal to your potential energy at the highest point:

$$\frac{mv^2}{2} = mgh$$

Once again we have a situation where the mass m can be canceled out from both sides of the equation. If we measure the height of your jump, this equation can be multiplied by two to get the square of your initial velocity:

$$v^2 = 2gh$$

Or, if we know your initial velocity, we can divide both sides of this equation to calculate how high you can jump:

$$h = \frac{v^2}{2g}$$

How High Can You Jump on Earth?

It's easy to measure how high you can jump on Earth. We don't need to calculate anything. Wikipedia has an interesting article, "Vertical Jump", that describes how to measure a person's ability to jump. The average height for males is in the neighborhood of 30 inches, or 2.5 feet. Since we know this for Earth's gravity, we can use the next-to-last equation above to calculate the initial velocity of your body as it departs the surface of the Earth:

$$v^2 = 2gh = 2 \times 32 \times 2.5 = 160 \text{ ft}^2/\text{sec}^2$$

Taking the square root of both sides,

$$v = 12.6 \text{ ft/sec}$$

How High Can You Jump on Mars?

If you jump on Mars, your legs should be able to provide the same amount of thrust as on Earth. Yes, your body weighs less on Mars, but the mass of your body is the same as on Earth. Newton's second law says the resulting acceleration is the force divided by the mass. The force is the same, the mass is the same, so the acceleration that produces your initial velocity will be the same. So your initial velocity in your upward jump will be the same on Mars as it is on Earth.

We can calculate how high you can jump on Mars by using the equation we derived above,

$$h = \frac{v^2}{2g}$$

since we know your initial velocity is the same as on Earth. But, wait! The acceleration due to gravity, g , is not the same on Mars. But if you go back to the beginning of this lesson, you'll remember we calculated the acceleration due to the gravitational force of a mass:

$$a = \frac{GM}{d^2}$$

In this case, we'll let M represent the mass of the planet Mars, and d represent your distance from the center of Mars. G continues to represent that universal gravitational constant discovered by Newton. The value of the acceleration a we are calculating here represents the value of g that applies to the surface of Mars. We can use the orbits of Deimos and Phobos to calculate the mass M of Mars. And the radius of Mars has been measured to be about 2100 miles, a little over half that of Earth. If we calculate the gravitational acceleration at the surface of Mars, we find it to be 0.39 times that of the Earth, or a little over a third of Earth's surface gravity:

$$g_{Mars} = 0.39g_{Earth} = 0.39 \times 32 \text{ ft/sec}^2 = 12.5 \text{ ft/sec}^2$$

Plugging this into the equation for how high you can jump on Mars,

$$h = \frac{v^2}{2g} = \frac{160 \text{ ft}^2/\text{sec}^2}{2 \times 12.5 \text{ ft/sec}^2} = 6.4 \text{ ft}$$

(Notice how the units of measure cancel out, leaving the desired unit for height? Physicists use this a lot as a check on their derivations.) Of course, this assumes you are not encumbered by your pressure suit. You shouldn't have any trouble executing a vertical jump of six feet inside the dome. This makes it very clear we will have to change the rules of basketball on Mars. We don't want Grandpa doing slam dunks.

This number was calculated for the average male jumper. A good athlete can jump 40 inches, and some have managed 50 inches. So a good athlete could jump about eight and a half feet.

Isn't it just a little bit amazing that I was able to tell you this without ever leaving Earth? The power of science.